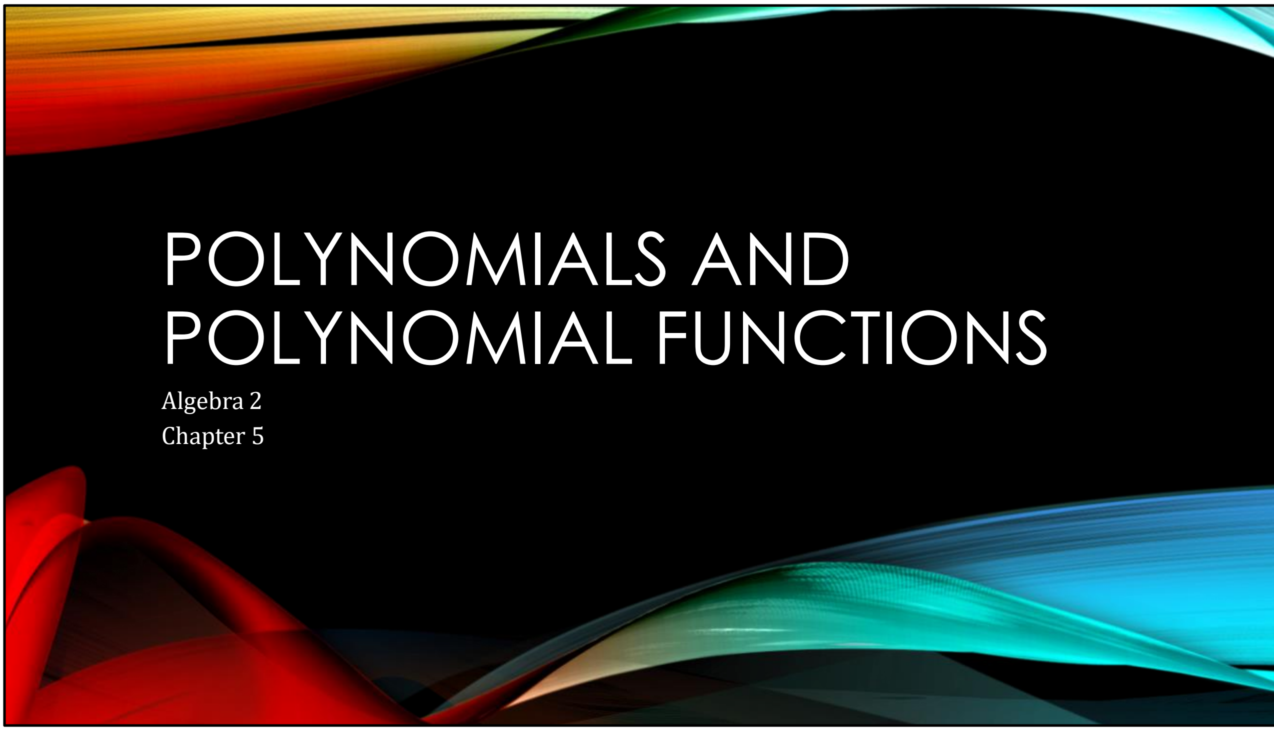

Algebra II 5

5.1 USE PROPERTIES OF EXPONENTS

Algebra 2

Chapter 5

- This Slideshow was developed to accompany the textbook
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    - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
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5.1 USE PROPERTIES OF EXPONENTS

- When numbers get very big or very small, such as the mass of the sun = 5.98×10^{30} kg or the size of a cell = 1.0×10^{-6} m, we use scientific notation to write the numbers in less space than they normally would take.

- The properties of exponents will help you understand how to work with scientific notation.

5.1 USE PROPERTIES OF EXPONENTS

- What is an exponent and what does it mean?
  - A superscript on a number.
  - It tells the number of times the number is multiplied by itself.

- Example;
  - x^3 = x x x

Base

Exponent

5.1 USE PROPERTIES OF EXPONENTS

- Properties of exponents
- $x^m \cdot x^n = x^{m+n}$ → product property (Multiply with same base, add exponents)
  - $x^2 \cdot x^3$
  - $= x^{2+3} = x^5$
- $(xy)^m = x^m y^m$ → power of a product property (Multiply in parentheses, distribute exponent)
  - $(2 \cdot x)^3$
  - $= 2^3 \cdot x^3$
  - $= 8x^3$
- $(x^m)^n = x^{mn}$ → power of a power property (Exponent-Parenthesis-Exponent, multiply exponents)
  - $(2^3)^4$
  - $= 2^{3 \cdot 4}$
  - $= 2^{12}$
  - $= 4096$

5.1 USE PROPERTIES OF EXPONENTS

- Properties of exponents (continued)
- $\frac{x^m}{x^n} = x^{m-n}$ → quotient property (Divide with same base, subtract exponents)
  - $\frac{x^4}{x^2}$
  - $= x^{4-2}$
  - $= x^2$
- $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$ → power of a quotient property (Divide in parentheses, distribute exponent)
  - $\left(\frac{4}{2}\right)^3$
  - $= \frac{4^3}{2^3}$
  - $= \frac{64}{8}$
  - $= 8$

$(x\ x)(x\ x\ x) = x^{2+3} = x^5$

$6^3 = 216$; $2^3 \cdot 3^3 = 8 \cdot 27 = 216$

$(2 \cdot 2 \cdot 2)^4 = 2^4 \cdot 2^4 \cdot 2^4 = 16 \cdot 16 \cdot 16 = 4096$; $(2^3)^4 = 2^{12} = 4096$

$(x\ x\ x\ x)/(x\ x) = x\ x = x^2$; $x^4 / x^2 = x^{4-2} = x^2$

$2^3 = 8$; $(4/2)^3 = 4^3/2^3 = 64/8 = 8$

5.1 USE PROPERTIES OF EXPONENTS

- $x^0 = 1$ → zero exponent property (Anything raised to 0 = 1)
- $x^{-m} = \frac{1}{x^m}$ → negative exponent property (Negative exponent, reciprocal base)
  - 2^3
    - $2 \cdot 2 \cdot 2 = 8$
  - 2^2
    - $2 \cdot 2 = 4$
  - 2^1
    - $2 = 2$
  - 2^0
    - $1 = 1$

Every time the exponent is reduced by 1, the result is divided by 2.

- 2^{-1}
  - $\frac{1}{2} = \frac{1}{2}$
- 2^{-2}
  - $\frac{1}{2 \cdot 2} = \frac{1}{4}$
- 2^{-3}
  - $\frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$

2 2 2 = 8
2 2 = 4
2 = 2
1
½
1/(2 2) = ¼
1/(2 2 2) = 1/8

5.1 USE PROPERTIES OF EXPONENTS

Simplify

- $5^{-4} \cdot 5^3$
  - Multiply with same base, add exponents
  - 5^{-4+3}
  - 5^{-1}
  - Negative exponent, reciprocal base
  - $\left(\frac{1}{5}\right)^1$
  - $\frac{1}{5}$
- $\left((-3)^2\right)^3$
  - Exponent-parenthesis-exponent, multiply exponents
  - $(-3)^{2\cdot 3}$
  - $(-3)^6$
  - 729

5.1 USE PROPERTIES OF EXPONENTS

Simplify

- $(3^2x^2y)^2$
  - Multiply in parentheses, distribute exponent
  - $3^{2\cdot 2}x^{2\cdot 2}y^2$
  - $3^4x^4y^2$
  - $81x^4y^2$

5.1 USE PROPERTIES OF EXPONENTS

Simplify

- $\frac{12x^5a^2}{2x^4} \cdot \frac{2a}{3a^2}$
  - Multiply across (same bases), add exponents
  - $\frac{(12 \cdot 2)(x^5)(a^2 \cdot a)}{(2 \cdot 3)(x^4)(a^2)}$
  - $\frac{24x^5a^3}{6x^4a^2}$
  - Divide (same bases), subtract exponents
  - $\left(\frac{24}{6}\right)\left(\frac{x^5}{x^4}\right)\left(\frac{a^3}{a^2}\right)$
  - $4xa$

- $\frac{5x^2y^{-3}}{8x^{-4}} \cdot \frac{4x^{-3}y^2}{10x^{-2}z^0}$
  - Multiply across (same bases, add exponents
  - $\frac{(5 \cdot 4)(x^2 \cdot x^{-3})(y^{-3} \cdot y^2)}{(8 \cdot 10)(x^{-4} \cdot x^{-2})(z^0)}$
  - $\frac{20x^{-1}y^{-1}}{80x^{-6}z^0}$
  - Anything with 0 exponent = 1
  - $\frac{20x^{-1}y^{-1}}{80x^{-6}}$
  - Divide (same bases), subtract exponents
  - $\left(\frac{20}{80}\right)\left(\frac{x^{-1}}{x^{-6}}\right)(y^{-1})$
  - $\frac{1x^5y^{-1}}{4}$
  - Negative exponent, reciprocal base
  - $\frac{x^5}{4y}$

5.1 USE PROPERTIES OF EXPONENTS

- To multiply or divide scientific notation
  - think of the leading numbers as the coefficients and the power of 10 as the base and exponent.
- Simplify
  - $2 \times 10^2 \cdot 5 \times 10^3$
    - Multiply (same base), add exponents
    - $2 \cdot 5 \times 10^{2+3}$
    - 10×10^5
    - 1×10^6

$10x10^{2+3} = 10x10^5 = 1x10^6$

HOMEWORK QUIZ

- 5.1 Homework Quiz

5.2 EVALUATE AND GRAPH POLYNOMIAL FUNCTIONS

Algebra 2

Chapter 5

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5.2 EVALUATE AND GRAPH POLYNOMIAL FUNCTIONS

- Large branches of mathematics spend all their time dealing with polynomials.
- They can be used to model many complicated systems.

5.2 EVALUATE AND GRAPH POLYNOMIAL FUNCTIONS

- Polynomial in one variable
  - Function that has one variable and there are powers of that variable and all the powers are positive
- $4x^3 + 2x^2 + 2x + 5$
- $100x^{1234} - 25x^{345} + 2x + 1$
- $\frac{2}{x}$
- $3xy^2$

Not Polynomials in one variable.

5.2 EVALUATE AND GRAPH POLYNOMIAL FUNCTIONS

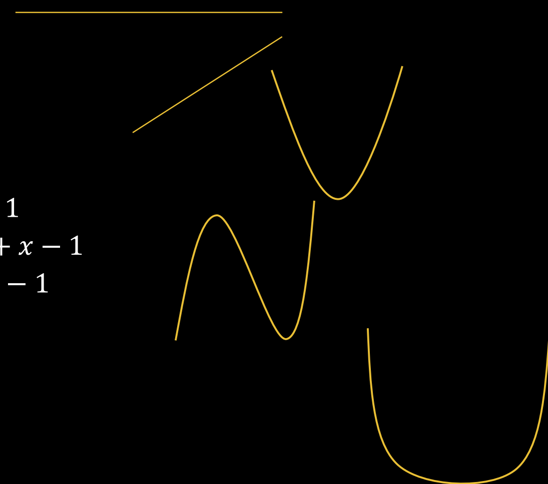
- Degree
  - Highest power of the variable

- What is the degree?
  - $4x^3 + 2x^2 + 2x + 5$
  - Highest exponent is 3, so the degree is 3.

3

5.2 EVALUATE AND GRAPH POLYNOMIAL FUNCTIONS

- Types of Polynomial Functions
- Degree → Type
  - 0 → Constant → $y = 2$
  - 1 → Linear → $y = 2x + 1$
  - 2 → Quadratic → $y = 2x^2 + x - 1$
  - 3 → Cubic → $y = 2x^3 + x^2 + x - 1$
  - 4 → Quartic → $y = 2x^4 + 2x^2 - 1$



5.2 EVALUATE AND GRAPH POLYNOMIAL FUNCTIONS

- Functions
  - $f(x) = 4x^3 + 2x^2 + 2x + 5$ means that this polynomial has the name f and the variable x
  - $f(x)$ does not mean f times x!
- Direct Substitution
  - Find $f(3)$
    - The x in $f(x)$ was replaced by the 3, so replace all the x's with 3 in the polynomial
    - $f(x) = 4x^3 + 2x^2 + 2x + 5$
    - $f(3) = 4(3)^3 + 2(3)^2 + 2(3) + 5$
    - $f(3) = 4(27) + 2(9) + 6 + 5$
    - $f(3) = 137$

ANS: the x was replaced by the 3, so in the polynomial replace the x with 3 and simplify.

$f(3) = 4(3)^3 + 2(3)^2 + 2(3) + 5 \Rightarrow 4(27) + 2(9) + 6 + 5 \Rightarrow 108 + 18 + 11 \Rightarrow 137$

5.2 EVALUATE AND GRAPH POLYNOMIAL FUNCTIONS

- Synthetic Substitution
  - Find $f(2)$ if $f(y) = -y^6 + 4y^4 + 3y^2 + 2y$
    - $f(y) = -y^6 + 0y^5 + 4y^4 + 0y^3 + 3y^2 + 2y + 0$

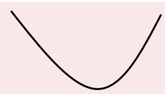
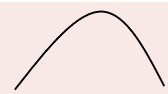
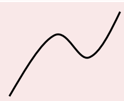
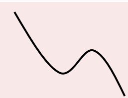
Coefficients with placeholders

2	-1	0	4	0	3	2	0
×		-2	-4	0	0	6	16
	-1	-2	0	0	3	8	16

- $f(2) = 16$

5.2 EVALUATE AND GRAPH POLYNOMIAL FUNCTIONS

- End Behavior
  - Polynomial functions always go towards ∞ or -∞ at either end of the graph

	Leading Coefficient +	**Leading Coefficient -**
Even Degree		
Odd Degree		

- Write
  - $f(x) \rightarrow \underline{+\infty}$ as $x \rightarrow -\infty$ and $f(x) \rightarrow \underline{+\infty}$ as $x \rightarrow +\infty$
  - (Only change the underlined portion)

(this is for even degree with positive first term)

5.2 EVALUATE AND GRAPH POLYNOMIAL FUNCTIONS

- Graphing polynomial functions
  - Make a table of values
  - Plot the points
  - Make sure the graph matches the appropriate end behavior

5.2 EVALUATE AND GRAPH POLYNOMIAL FUNCTIONS

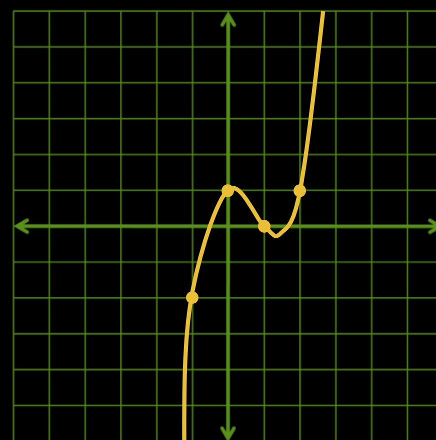
Graph $f(x) = x^3 - 2x^2 + 1$

- End behavior
  - Odd degree, positive leading coefficient

$f(x)$ → -∞ as x → -∞ and $f(x)$ → +∞ as x → +∞

- Make a table of values
- Plot points
- Connect with a smooth curve
- Make sure it matches end behavior

x	*y*
-3	-44
-2	-15
-1	-2
0	1
1	0
2	1
3	10



HOMEWORK QUIZ

- 5.2 Homework Quiz

5.3 ADD, SUBTRACT, AND MULTIPLY POLYNOMIALS

Algebra 2

Chapter 5

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5.3 ADD, SUBTRACT, AND MULTIPLY POLYNOMIALS

- Adding, subtracting, and multiplying are always good things to know how to do.
- Sometimes you might want to combine two or more models into one big model.

5.3 ADD, SUBTRACT, AND MULTIPLY POLYNOMIALS

- Adding and subtracting polynomials
  - Add or subtract the coefficients of the terms with the same power.
  - Called combining like terms.
- Add or Subtract
  - $(5x^2 + x - 7) + (-3x^2 - 6x - 1)$
    - $\left(5x^2 + (-3x^2)\right) + \left(x + (-6x)\right) + \left(-7 + (-1)\right)$
    - $2x^2 - 5x - 8$
  - $(3x^3 + 8x^2 - x - 5) - (5x^3 - x^2 + 17)$
    - $(3x^3 - 5x^3) + \left(8x^2 - (-x^2)\right) + (-x) + (-5 - 17)$
    - $-2x^3 + 9x^2 - x - 22$

$2x^2 - 5x - 8$

$-2x^3 + 9x^2 - x - 22$

5.3 ADD, SUBTRACT, AND MULTIPLY POLYNOMIALS

- Multiplying polynomials
  - Use the distributive property
- Multiply
  - $(x-3)(x+4)$
  - $x(x+4)+(-3)(x+4)$
  - $x^2+4x+(-3x)+(-12)$
  - x^2+x-12

- $(x+2)(x^2+3x-4)$
  - $x(x^2+3x-4)+2(x^2+3x-4)$
  - $x^3+3x^2-4x+2x^2+6x-8$
  - x^3+5x^2+2x-8

5.3 ADD, SUBTRACT, AND MULTIPLY POLYNOMIALS

- $(x-1)(x+2)(x+3)$
  - Multiply the 1st two parts
  - $\Big((x(x+2)) + (-1(x+2))\Big)(x+3)$
  - $(x^2+2x-1x-2)(x+3)$
  - $(x^2+x-2)(x+3)$
  - Multiply the result with the 3rd part
  - $x^2(x+3)+x(x+3)-2(x+3)$
  - $x^3+3x^2+x^2+3x-2x-6$
  - x^3+4x^2+x-6

5.3 ADD, SUBTRACT, AND MULTIPLY POLYNOMIALS

- Special Product Patterns
  - Sum and Difference
    - $(a-b)(a+b) = a^2 - b^2$
  - Square of a Binomial
    - $(a \pm b)^2 = a^2 \pm 2ab + b^2$
  - Cube of a Binomial
    - $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

5.3 ADD, SUBTRACT, AND MULTIPLY POLYNOMIALS

- Simplify
- $(x+2)^3$
  - Cube of Binomial
  - $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
  - $(x+2)^3 = x^3 + 3x^2 2 + 3x 2^2 + 2^3$
  - $= x^3 + 6x^2 + 12x + 8$
- $(x-3)^2$
  - Square of Binomial
  - $(a-b)^2 = a^2 - 2ab + b^2$
  - $(x-3)^2 = x^2 - 2x3 + 3^2$
  - $= x^2 - 6x + 9$

HOMEWORK QUIZ

- 5.3 Homework Quiz

5.4 FACTOR AND SOLVE POLYNOMIAL EQUATIONS

Algebra 2

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5.4 FACTOR AND SOLVE POLYNOMIAL EQUATIONS

- A manufacturer of shipping cartons who needs to make cartons for a specific use often has to use special relationships between the length, width, height, and volume to find the exact dimensions of the carton.
- The dimensions can usually be found by writing and solving a polynomial equation.
- This lesson looks at how factoring can be used to solve such equations.

5.4 FACTOR AND SOLVE POLYNOMIAL EQUATIONS

How to Factor

1. Greatest Common Factor
   - Comes from the distributive property
   - If the same number or variable is in each of the terms, you can bring the number to the front times everything that is left.
   - $3x^2y + 6xy - 9xy^2$
     - $3xy(x + 2 - 3y)$
   - Look for this first!

3xy(x + 2 – y)

5.4 FACTOR AND SOLVE POLYNOMIAL EQUATIONS

2. Check to see how many terms
   - Two terms
     - Difference of two squares: $a^2 - b^2 = (a - b)(a + b)$
       - $9x^2 - y^4$
       - $(3x)^2 - (y^2)^2$
       - $(3x - y^2)(3x + y^2)$
     - Sum of Two Cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
       - $8x^3 + 27$
       - $(2x)^3 + 3^3$
       - $(2x + 3)\left((2x)^2 - 2x(3) + 3^2\right)$
       - $(2x + 3)(4x^2 - 6x + 9)$
     - Difference of Two Cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
       - $y^3 - 8$
       - $y^3 - 2^3$
       - $(y - 2)(y^2 + 2y + 4)$

$(3x - y^2)(3x + y^2)$
$(2x + 3)(4x^2 - 6x + 9)$
$(y - 2)(y^2 + 2y + 4)$

5.4 FACTOR AND SOLVE POLYNOMIAL EQUATIONS

- Three terms
  - General Trinomials → $ax^2 + bx + c$

1. Write two sets of parentheses (   )(   )
2. Guess and Check
3. The Firsts multiply to make ax^2
4. The Lasts multiply to make c
5. The Outers + Inners make bx

- $x^2 + 7x + 10$
  - $(x + 5)(x + 2)$
  - Check Outers + Inners = bx
    - $2x + 5x = 7x$ ✓
- $6x^2 - 7x - 20$
  - $(3x + 4)(2x - 5)$
  - Check Outers + Inners = bx
    - $-15x + 8x = -7x$ ✓

5.4 FACTOR AND SOLVE POLYNOMIAL EQUATIONS

- Four terms
  - Grouping
    - Group the terms into sets of two so that you can factor a common factor out of each set
    - Then factor the factored sets (Factor twice)
    - $b^3 - 3b^2 - 2b + 6$
      - $(b^3 - 3b^2) + (-2b + 6)$
      - $b^2(b - 3) - 2(b - 3)$
      - $(b - 3)(b^2 - 2)$

5.4 FACTOR AND SOLVE POLYNOMIAL EQUATIONS

3. Try factoring more!

- Factor
  - $a^2x - b^2x + a^2y - b^2y$
  - No common factor, four terms so factor by grouping
  - $(a^2x - b^2x) + (a^2y - b^2y)$
  - $x(a^2 - b^2) + y(a^2 - b^2)$
  - $(a^2 - b^2)(x + y)$
  - Factor more, difference of squares
  - $(a - b)(a + b)(x + y)$

$x(a^2 - b^2) + y(a^2 - b^2) = (x + y)(a^2 - b^2) = (x + y)(a - b)(a + b)$

5.4 FACTOR AND SOLVE POLYNOMIAL EQUATIONS

- $3a^2z - 27z$
  - Common factor
  - $3z(a^2 - 9)$
  - Factor more, difference of squares
  - $3z(a - 3)(a + 3)$
- $n^4 - 81$
  - Common factor (there's none)
  - Count terms (two, so look for special patterns)
  - Difference of squares
  - $(n^2)^2 - 9^2$
  - $(n^2 - 9)(n^2 + 9)$
  - Factor more (first part is difference of squares)
  - $(n - 3)(n + 3)(n^2 + 9)$

$3z(a^2 - 9) = 3z(a - 3)(a + 3)$
$(n^2 - 9)(n^2 + 9) = (n^2 + 9)(n - 3)(n + 3)$

5.4 FACTOR AND SOLVE POLYNOMIAL EQUATIONS

- Solving Equations by Factoring
  - Make = 0
  - Factor
  - Make each factor = 0 because if one factor is zero, 0 time anything = 0

5.4 FACTOR AND SOLVE POLYNOMIAL EQUATIONS

Solve

- $2x^5 = 18x$
- Make = 0
- $2x^5 - 18x = 0$
- Common factor
- $2x(x^4 - 9) = 0$
- Factor more (difference of squares)
- $2x(x^2 - 3)(x^2 + 3) = 0$

- Each factor = 0
- $2x = 0$ $\quad x^2 - 3 = 0$ $\quad x^2 + 3 = 0$
- $x = 0$ $\quad x^2 = 3$ $\quad x^2 = -3$
- $\quad x = \pm\sqrt{3}$ $\quad x = \pm\sqrt{3}i$

HOMEWORK QUIZ

- 5.4 Homework Quiz

5.5 APPLY THE REMAINDER AND FACTOR THEOREMS

Algebra 2

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5.5 APPLY THE REMAINDER AND FACTOR THEOREMS

- So far we done add, subtracting, and multiplying polynomials.
- Factoring is similar to division, but it isn't really division.
- Today we will deal with real polynomial division.

5.5 APPLY THE REMAINDER AND FACTOR THEOREMS

- Long Division
  - Done like long division with numbers
- $\frac{y^4+2y^2-y+5}{y^2-y+1}$

Divide the 1st terms

Multiply

Subtract

- $$\begin{array}{r} y^2 + y + 2 + \frac{3}{y^2-y+1} \\ y^2 - y + 1 \overline{)\, y^4 + 0y^3 + 2y^2 - y + 5} \\ \underline{y^4 - y^3 + y^2} \qquad\quad \\ y^3 + y^2 - y \quad \\ \underline{y^3 - y^2 + y} \quad \\ 2y^2 - 2y + 5 \\ \underline{2y^2 - 2y + 2} \\ 3 \end{array}$$

5.5 APPLY THE REMAINDER AND FACTOR THEOREMS

- $\frac{x^3+4x^2-3x+10}{x+2}$

- $$\begin{array}{r} x^2 + 2x - 7 + \frac{24}{x+2} \\ x+2\overline{)x^3 + 4x^2 - 3x + 10} \\ \underline{x^3 + 2x^2} \qquad\qquad \\ 2x^2 - 3x \qquad \\ \underline{2x^2 + 4x} \qquad \\ -7x + 10 \\ \underline{-7x - 14} \\ 24 \end{array}$$

Divide the 1st terms

Multiply

Subtract

5.5 APPLY THE REMAINDER AND FACTOR THEOREMS

- Synthetic Division
  - Shortened form of long division for dividing by a **binomial**
  - Only when dividing by $(x - r)$

5.5 APPLY THE REMAINDER AND FACTOR THEOREMS

- Synthetic Division
  - Like synthetic substitution
  - $(-5x^5 - 21x^4 - 3x^3 + 4x^2 + 2x + 2)/(x + 4)$

Coefficients with placeholders

-4	-5	-21	-3	4	2	2
×		20	4	-4	0	-8
	-5	-1	1	0	2	-6

- The bottom row give the coefficients of the answer. This is called the depressed polynomial.
- The number in the box is the remainder.
- Start with an exponent one less than the original expression since you divided by *x*.

$$-5x^4 - x^3 + x^2 + 2 + \frac{-6}{x+4}$$

5.5 APPLY THE REMAINDER AND FACTOR THEOREMS

- $(y^5 + 32)(y + 2)^{-1}$

Coefficients with placeholders

-2	1	0	0	0	0	32
		-2	4	-8	16	-32
	1	-2	4	-8	16	0

- The degree was 5, now the depressed polynomial is one less degree. It's degree is 4.

$$y^4 - 2y^3 + 4y^2 - 8y + 16$$

5.5 APPLY THE REMAINDER AND FACTOR THEOREMS

- Remainder Theorem
  - If polynomial $f(x)$ is divided by the binomial $(x - a)$, then the remainder equals $f(a)$.
  - Synthetic substitution

5.5 APPLY THE REMAINDER AND FACTOR THEOREMS

- The Factor Theorem
  - The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$.
  - In other words, it's a factor if the remainder is 0.

5.5 APPLY THE REMAINDER AND FACTOR THEOREMS

- Using the factor theorem, you can find the factors (and zeros) of polynomials
- Simply use synthetic division using your first zero (you get these off of problem or off of the graph where they cross the *x*-axis)
- The polynomial answer is one degree less and is called the depressed polynomial.
- Divide the depressed polynomial by the next zero and get the next depressed polynomial.
- Continue doing this until you get to a quadratic which you can factor or use the quadratic formula to solve.

5.5 APPLY THE REMAINDER AND FACTOR THEOREMS

- Show that $x-2$ is a factor of $x^3+7x^2+2x-40$. Then find the remaining factors.
- Start with synthetic division for the first factor.

2	1	7	2	-40
		2	18	40
	1	9	20	0

- Since the remainder is 0, $x-2$ is a factor.
- The depressed polynomial is the original function without the factor $x-2$. The other factors are still there. If we factor the result we get the other factors.
- Depressed polynomial
  - $x^2+9x+20$
  - $(x+4)(x+5)$
- All the factors are $(x-2)(x+4)(x+5)$

All factors are (x + 4)(x + 5)(x – 2)

HOMEWORK QUIZ

- 5.5 Homework Quiz

5.6 FIND RATIONAL ZEROS

Algebra 2

Chapter 5

- This Slideshow was developed to accompany the textbook
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5.6 FIND RATIONAL ZEROS

- Rational Zero Theorem
  - Given a polynomial function, the rational zeros will be in the form of $\frac{p}{q}$ where p is a factor of the last (or constant) term and q is the factor of the leading coefficient.

5.6 FIND RATIONAL ZEROS

- List all the possible rational zeros of $f(x) = 2x^3 + 4x^2 - 3x + 9$
- p are factors of the constant term, 9
  - $p = \pm 1, \pm 3, \pm 9$
- q are factors of the leading coefficient, 2
  - $q = \pm 1, \pm 2$
- The possible rational zeros are all the p over all the q
  - $\frac{p}{q} = \pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{9}{1}, \pm\frac{9}{2}$
  - Simplify and put in order from smallest to greatest
  - $\frac{p}{q} = \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 3, \pm\frac{9}{2}, \pm 9$

5.6 FIND RATIONAL ZEROS

- Find all rational zeros of $f(x) = x^3 - 4x^2 - 2x + 20$.
- List the possible rational zeros by listing the p and q.
  - $p = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$
  - $q = \pm 1$
  - $\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$
- Pick one to try with synthetic division.

1	1	-4	-2	20
		1	-3	-5
	1	-3	-5	15

- The remainder is not zero, so try another.
- To help graph the function on a graphing calculator and find the x-intercepts.

-2	1	-4	-2	20
		-2	12	-20
	1	-6	10	0

- This is a zero!
- The depressed polynomial will be quadratic, so we can factor it or use the quadratic formula.
- $x^2 - 6x + 10$
- $x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$
- $x = \frac{6 \pm \sqrt{-4}}{2} = \frac{6 \pm 2i}{2} = 3 \pm i$
- These are not rational, so rational zeros are -2

ANS: List possible rational roots; ±1, ±2, ±4, ±5, ±10, ±20
If there are many zeros you many want to graph to choose which roots to try
Lets look for the negative first. Use a table a shortened form of synthetic division
Since the remainder was zero -2 is a root and the depressed polynomial is $x^2 - 6x + 10$
Repeat the process on the depressed polynomial until you get a quadratic for the depressed polynomial then use the quadratic formula
$x = 3 \pm i, -2$

HOMEWORK QUIZ

- 5.6 Homework Quiz

5.7 APPLY THE FUNDAMENTAL THEOREM OF ALGEBRA

Algebra 2

Chapter 5

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5.7 APPLY THE FUNDAMENTAL THEOREM OF ALGEBRA

- When you are finding the zeros, how do you know when you are finished?
- Today we will learn about how many zeros there are for each polynomial function.

5.7 APPLY THE FUNDAMENTAL THEOREM OF ALGEBRA

- Fundamental Theorem of Algebra
  - A polynomial function of degree greater than zero has at least one zero.
  - These zeros may be imaginary however.
  - There is the same number of zeros as there is degree. You may have the same zero more than once though.
    - $x^2 + 6x + 9 = 0$
    - $(x+3)(x+3) = 0$
    - $x + 3 = 0 \qquad x + 3 = 0$
    - $x = -3 \qquad x = -3$
    - Zeros are -3 and -3

5.7 APPLY THE FUNDAMENTAL THEOREM OF ALGEBRA

- Complex Conjugate Theorem
  - If the complex number $a + bi$ is a zero, then $a - bi$ is also a zero.
  - Complex zeros come in pairs.

- Irrational Conjugate Theorem
  - If $a + \sqrt{b}$ is a zero, then so is $a - \sqrt{b}$

5.7 APPLY THE FUNDAMENTAL THEOREM OF ALGEBRA

- Find the zeros of $f(x) = x^3 - 7x^2 + 16x - 10$.
- Find the $\frac{p}{q}$
  - $p = \pm 1, \pm 2, \pm 5, \pm 10$
  - $q = \pm 1$
  - $\frac{p}{q} = \pm 1, \pm 2, \pm 5, \pm 10$
- Pick one and try it with synthetic division

1	1	-7	16	-10
		1	-6	10
	1	-6	10	0

- This is a zero!
- The depressed polynomial will be quadratic, so we can factor it or use the quadratic formula.
- $x^2 - 6x + 10$
- $x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$
- $x = \frac{6 \pm \sqrt{-4}}{2}$
- $x = \frac{6 \pm 2i}{2}$
- $x = 3 \pm i$
- The zeros are $1, 3 + i, 3 - i$

Find p's, q's, and p/q
Choose a p/q (1 works well)
Use synthetic division to check to see if it is a factor (it is)
The depressed polynomial is a quadratic, so use the quadric formula to solve.
The zeros are 1, 3 ± i

5.7 APPLY THE FUNDAMENTAL THEOREM OF ALGEBRA

- Write a polynomial function that has the given zeros. $2, 4i$
- Since $4i$ is a zero, the conjugate $-4i$ is also a zero.
- Write factors using the zeros in the form $(x - a)$.
  - $(x-2)(x-4i)(x+4i)$
- Multiply the complex conjugates
  - $(x-2)(x^2+4ix-4ix-16i^2)$
  - $(x-2)\left(x^2-16(-1)\right)$
  - $(x-2)(x^2+16)$
- Multiply these
  - $x^3+16x-2x^2-32$
  - $x^3-2x^2+16x-32$

ANS: $(x – 2)(x – 4i)(x + 4i)$ Don’t forget -4i is also a root.
$(x – 2)(x^2 – 16i^2) \rightarrow (x – 2)(x^2 + 16) \rightarrow x^3 - 2x^2 + 16x - 32$

5.7 APPLY THE FUNDAMENTAL THEOREM OF ALGEBRA

- Descartes' Rule of Signs
  - If $f(x)$ is a polynomial function, then
    - The number of **positive** real zeros is equal to the number of sign changes in $\boldsymbol{f(x)}$ or less by even number.
    - The number of **negative** real zeros is equal to the number of sign changes in $\boldsymbol{f(-x)}$ or less by even number.

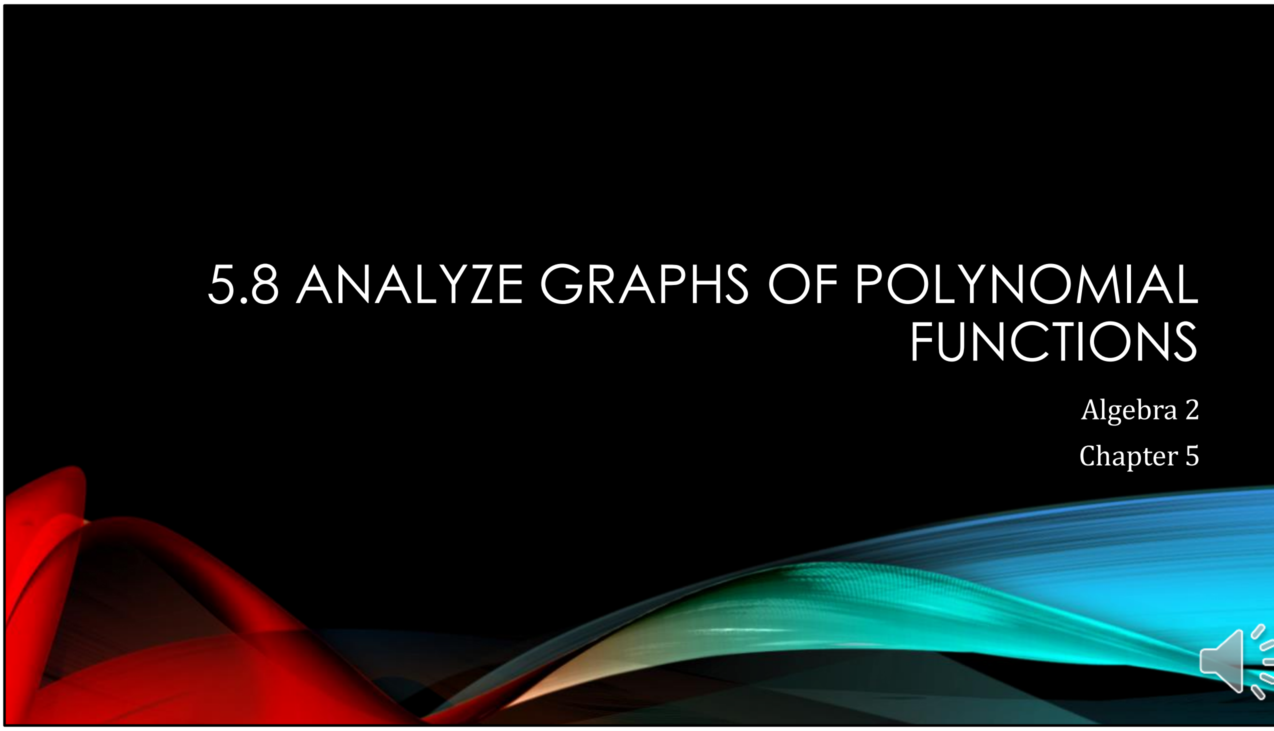
5.7 APPLY THE FUNDAMENTAL THEOREM OF ALGEBRA

- Determine the possible number of positive real zeros, negative real zeros, and imaginary zeros for $g(x) = 2x^4 - 3x^3 + 9x^2 - 12x + 4$
  - Positive zeros: $g(x) = 2x^4 - 3x^3 + 9x^2 - 12x + 4$
    - 4, 2, or 0
  - Negative zeros: $g(-x) = 2(-x)^4 - 3(-x)^3 + 9(-x)^2 - 12(-x) + 4$
    - $g(-x) = 2x^4 + 3x^3 + 9x^2 + 12x + 4$
    - 0

Positive	Negative	Imaginary	Total
4	0	0	4
2	0	2	4
0	0	4	4

HOMEWORK QUIZ

- 5.7 Homework Quiz


5.8 ANALYZE GRAPHS OF POLYNOMIAL FUNCTIONS
Algebra 2
Chapter 5

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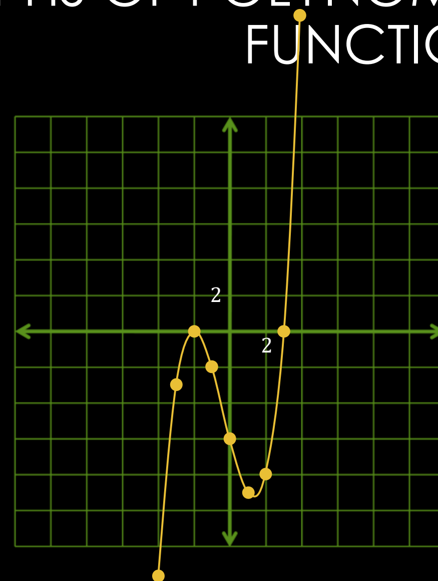
5.8 ANALYZE GRAPHS OF POLYNOMIAL FUNCTIONS

- If we have a polynomial function, then
  - k is a zero or root
  - k is a solution of $f(x) = 0$
  - k is an x-intercept if k is a real number
  - $x - k$ is a factor of $f(x)$

5.8 ANALYZE GRAPHS OF POLYNOMIAL FUNCTIONS

- Use x-intercepts to graph a polynomial function
- $f(x) = \frac{1}{2}(x+2)^2(x-3)$
  - Since $(x+2)$ and $(x-3)$ are factors of the polynomial, the x-intercepts are -2 and 3
  - Plot the x-intercepts
  - Create a table of values to finish plotting points around the x-intercepts
  - Draw a smooth curve through the points

x	-4	-3	-2	-1	0	1	2	3	4
y	-14	-3	0	-2	-6	-9	-8	0	18

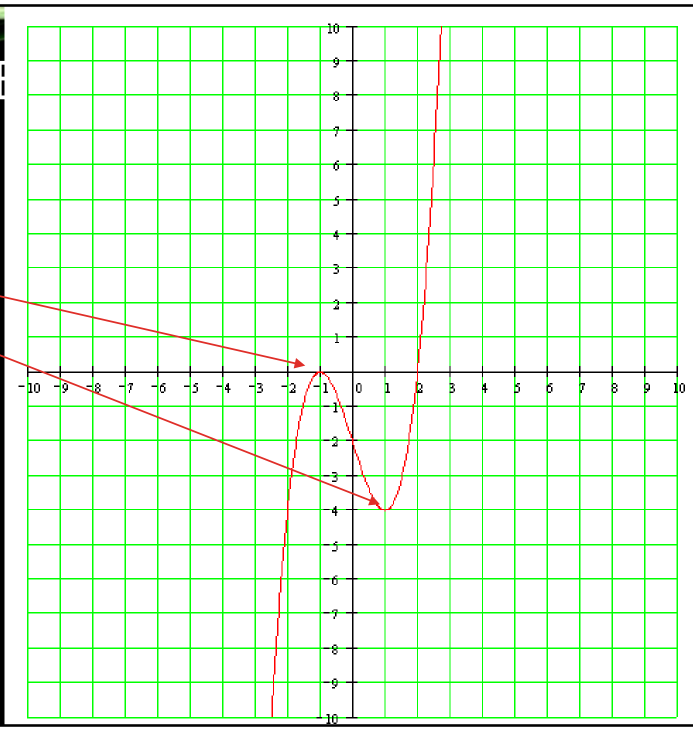


5.8 ANALYZE GRAPHS OF POLYNOMIAL FUNCTIONS

- Turning Points
  - Local Maximum and minimum (turn from going up to down or down to up)
  - The graph of every polynomial function of degree n can have at most $n-1$ turning points.
  - If a polynomial function has n distinct real zeros, the function will have exactly $n-1$ turning points.
  - Calculus lets you find the turning points easily.

5.8 ANALYZE

- What are the turning points?
  - Local maximum at $(-1, 0)$
  - Local minimum at $(1, -4)$



HOMEWORK QUIZ

- 5.8 Homework Quiz

5.9 WRITE POLYNOMIAL FUNCTIONS AND MODELS

Algebra 2

Chapter 5

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5.9 WRITE POLYNOMIAL FUNCTIONS AND MODELS

- You keep asking, "Where will I ever use this?" Well today we are going to model a few situations with polynomial functions.

5.9 WRITE POLYNOMIAL FUNCTIONS AND MODELS

- Writing a function from the x-intercepts and one point
  - Write the function as factors with an a in front
  - $y = a(x-p)(x-q)\ldots$
  - Use the other point to find a

- Write a polynomial function with x-intercepts $-2, 1, 3$ and passes through $(0, 2)$
- Write a factored function with a
  - $y = a(x+2)(x-1)(x-3)$
- Plug in the point for x and y
  - $2 = a(0+2)(0-1)(0-3)$
  - $2 = a(6)$
  - $\frac{1}{3} = a$
- Write the function
  - $y = \frac{1}{3}(x+2)(x-1)(x-3)$

ANS: y = a(x + 2)(x – 1)(x – 3)

2 = a(0 + 2)(0 – 1)(0 – 3) → 2 = 6a → a = 1/3

y = 1/3 (x + 2)(x – 1)(x – 3)

5.9 WRITE POLYNOMIAL FUNCTIONS AND MODELS

- Show that the n^{th}-order differences for the given function of degree n are nonzero and constant.
  - Find the values of the function for equally spaced intervals
  - Find the differences of the y values
  - Find the differences of the differences and repeat until all are the same value

5.9 WRITE POLYNOMIAL FUNCTIONS AND MODELS

- Show that the 3rd order differences are constant of $f(x) = 2x^3 + x^2 + 2x + 1$

x	0		1		2		3		4		5
y	1		6		25		70		153		286
1st		5		19		45		83		133	
2nd			14		26		38		50		
3rd				12		12		12			

- Because the 3rd order differences were the same, the function is degree 3.

5.9 WRITE POLYNOMIAL FUNCTIONS AND MODELS

- Finding a model given several points
  - Find the degree of the function by finding the finite differences
    - Degree = order of constant nonzero finite differences
  - Write the basic standard form functions (i.e. $f(x) = ax^3 + bx^2 + cx + d$)
  - Fill in x and $f(x)$ with the points
  - Use some method to find a, b, c, and d
    - Cramer's rule or graphing calculator using matrices or computer program

5.9 WRITE POLYNOMIAL FUNCTIONS AND MODELS

- Find a polynomial function to fit:
- $f(1) = -2, f(2) = 2, f(3) = 12, f(4) = 28, f(5) = 50, f(6) = 78$
- Use the n^{th} order differences to find the degree.

x	1		2		3		4		5		6
y	-2		2		12		28		50		78
1^{st}		4		10		16		22		28	
2^{nd}			6		6		6		6		

- Since the 2^{nd} order differences are constant, the function is degree 2.
- $y = ax^2 + bx + c$

Continued on next slide…

ANS: Find finite differences

1^{st} order: 2 - -2 = **4**, 12 – 2 = **10**, 28 – 12 = **16**, 50 – 28 = **22**, 78 – 50 = **28**

2^{nd} order: 10 – 4 = **6**, 16 – 10 = **6,** 22 – 16 = **6**, 28 – 22 = **6**

degree = 2

$f(x) = ax^2 + bx + c$

$-2 = a(1)^2 + b(1) + c$

$2 = a(2)^2 + b(2) + c$

$12 = a(3)^2 + b(3) + c$

$f(x) = 3x^2 – 5x$

5.9 WRITE POLYNOMIAL FUNCTIONS AND MODELS

- Continuation of find a polynomial function to fit:
- $f(1) = -2, f(2) = 2, f(3) = 12, f(4) = 28, f(5) = 50, f(6) = 78$
- $y = ax^2 + bx + c$
- Plug in 3 points because there are 3 letters other than x and y.
  - $-2 = a(1)^2 + b(1) + c$
  - $2 = a(2)^2 + b(2) + c$
  - $12 = a(3)^2 + b(3) + c$
- Simplifying gives
  - $a + b + c = -2$
  - $4a + 2b + c = 2$
  - $9a + 3b + c = 12$

- Solve like you did back in chapter 3 (I'm using an inverse matrix on my calculator.)

- $a = 3, b = -5, c = 0$
- $f(x) = 3x^2 - 5x$

5.9 WRITE POLYNOMIAL FUNCTIONS AND MODELS

- Regressions on TI Graphing Calculator

1. Push STAT ↓ Edit…
2. Clear lists, then enter *x*'s in 1st column and *y*'s in 2nd
3. Push STAT → CALC ↓ (regression of your choice)
4. Push ENTER
5. If you get a new window make sure X are in L1 and Y are in L2, then press CALCULATE. If you didn't get a new window just push ENTER again
6. Read your answer

5.9 WRITE POLYNOMIAL FUNCTIONS AND MODELS

- Regressions using Microsoft Excel

1. Enter x's and y's into 2 columns
2. Insert X Y Scatter Chart
3. In Chart Tools: Layout pick Trendline → More Trendline options
4. Pick a Polynomial trendline and enter the degree of your function AND pick Display Equation on Chart
5. Click Done
6. Read your answer off of the chart.

HOMEWORK QUIZ

- 5.9 Homework Quiz